Breakdown of Hydrodynamics in the Radial Breathing Mode
of a Strongly-Interacting Fermi Gas

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We measure the magnetic field dependence of the frequency and damping time for the radial breathing mode of an optically trapped, Fermi gas of ^{6}Li atoms near a Feshbach resonance. The measurements address the apparent discrepancy between the results of Kinast et al., [Phys. Rev. Lett. **92**, 150402 (2004)] and those of Bartenstein et al., [Phys. Rev. Lett. **92**, 203201 (2004)]. Over the range of magnetic field from 770 G to 910 G, the measurements confirm the results of Kinast et al. Close to resonance, the measured frequencies are in excellent agreement with predictions for a unitary hydrodynamic gas. At a field of 925 G, the measured frequency begins to decrease below predictions. For fields near 1080 G, we observe a breakdown of hydrodynamic behavior, which is manifested by a sharp increase in frequency and damping rate. The observed breakdown is in qualitative agreement with the sharp transition observed by Bartenstein et al., at 910 G.

Optically trapped, strongly-interacting Fermi gases of atoms [1, 2] are possibly the most convenient and flexible systems for exploring the rich physics of the BEC-BCS crossover [3], i.e., the transitional regime between Bose and Fermi statistics. Atomic systems are unique in that the state of the gas can be tuned throughout the whole crossover region, simply by varying a magnetic field. The last two years have seen explosive progress in experiments on degenerate, strongly-interacting Fermi gases which include anisotropic expansion [2], studies of universal interactions [2, 4, 5], and molecular condensates on the BEC side of the crossover [4, 6, 7, 8]. Recently, microscopic evidence for superfluidity has been obtained by observing preformed pairs on the BCS side [9, 10] and by measurement of the gap [11, 12]. The focus of this Rapid Communication is on extension and verification of experiments which probe the macroscopic properties of the trapped gas by studying the breathing mode [13, 14]. Mapping the frequency of the mode throughout the BEC-BCS crossover, as a function of the magnetic field, tests the predictions for the equation of state and can be used to verify current many-body calculational methods.

Kinast et al. [13], have measured the frequencies and damping times of the radial breathing mode in an optically trapped gas of ^{6}Li near a Feshbach resonance [13]. They find that for magnetic fields in the range of 770-910 G, the frequencies are close to the hydrodynamic predictions [15, 16, 17, 18, 19, 20] and that the damping rate drops rapidly as the temperature is lowered. The temperature dependence of the damping rate is consistent with a transition to a superfluid state at low temperature and inconsistent with a picture of a collisional normal fluid [21, 22, 23]. Bartenstein et al., also have measured the magnetic field dependence of the frequencies for both the axial and radial breathing modes of an optically trapped gas of ^{6}Li [14]. For the axial mode, the agreement with predictions for a hydrodynamic gas

is quite good [17, 20, 24] and the minimum damping rate is very low. In contrast, the measured frequencies for the radial mode are 9% below the predictions of hydrodynamics, and both the frequency of the radial breathing mode and the damping rate exhibit an abrupt increase at a magnetic field near 910 G. Further, the damping rate measured near 910 G by Bartenstein et al., exceeds the maximum damping rate allowed in a collisional gas by more than a factor of 5, signaling a possible transition between a superfluid state and a normal gas.

The discrepancies between the two groups in the measured frequencies and damping rates of the radial breathing mode motivated an additional study of the magnetic field dependence. In this paper, we describe new measurements of the magnetic field dependence of the radial breathing mode in an optically trapped, resonantly interacting gas of ^{6}Li, over a magnetic field range from 750-1114 G (the widest range accessible to us at present). For fields below 950 G, we find that the measurements confirm the results obtained by Kinast et al., [13]. For fields near 1080 G, we observe a breakdown of hydrodynamic behavior, which is manifested in a sharp increase in frequency and damping rate. The observed breakdown is in qualitative agreement with the sharp transition observed by Bartenstein et al., [14], which occurred at 910 G.

In the measurements, we prepare a highly degenerate 50-50 mixture of the two lowest spin states of ^{6}Li atoms in an ultrastable CO_2 laser trap [25], using forced evaporation near a Feshbach resonance [2]. The trap depth is lowered by a factor of $\simeq 580$ over 4 s, then recompressed to 4.6% of the full trap depth in 1 s and held for 0.5 s to assure equilibrium. The number of atoms is determined from the column density obtained by absorption imaging on a two-level state-selective cycling transition [2, 13]. In the measurements of the cloud column density, we take optical saturation into account exactly and arrange to have very small optical pumping out of the two-level system. The column density is integrated in the axial direction and divided by the total atom number to obtain the one dimensional density $n(x)$, which is normalized to 1. Fitting the measured distributions of the expanded cloud

with a Thomas-Fermi distribution yields the cloud radius and the reduced temperature $(T/T_F)_{fit} \simeq 0.1$, where T_F is the Fermi temperature.

To excite the transverse breathing mode, the trap is turned off abruptly ($\leq 1\,\mu$s) and turned back on after a delay of $t_0 = 25\,\mu$s. Then the sample is held for a variable time t_{hold}. Finally, the trap is extinguished suddenly, releasing the gas which is imaged after 1 ms. The temperature increase from the excitation is esitmated as $\Delta T/T_F \leq 0.015$ when the gas thermalizes [13].

To determine the breathing mode frequency and damping time, the transverse spatial distribution of the cloud is fit with a one dimensional zero-temperature Thomas-Fermi (T-F) profile to extract the radius, σ_{TF} as a function of t_{hold}. To acquire each curve, 60-90 equally spaced values of t_{hold} are chosen in the time range of interest. The chosen values of t_{hold} are randomly ordered during data acquisition to avoid systematic error. Three full sequences are obtained and averaged. The averaged data is fit with a damped sinusoid $x_0 + A \exp(-t_{hold}/\tau) \sin(\omega\, t_{hold} + \varphi)$. For a $t_0 = 25\,\mu$s excitation time, we observe damping times up to $\tau = 7$ ms for both the interacting and noninteracting gas at 4.6% of full trap depth.

We normalize the breathing mode frequencies to the transverse oscillation frequency $\omega_\perp$ for the noninteracting gas at 526 G, where the scattering length is nearly zero [26, 27]. Parametric resonance measurements yield the trap oscillation frequencies (uncorrected for anharmonicity) $\omega_x = 2\pi \times 1600(10)$ Hz, $\omega_y = 2\pi \times 1500(10)$ Hz and $\omega_z = 2\pi \times 70(5)$ Hz at 4.6% of full trap depth. As in our previous study, the frequency ω_x is also calibrated by exciting the breathing mode in a noninteracting sample, which yields agreement to $\leq 1\%$. In addition, the measured breathing mode frequency $2\omega_x$ is used to scale the value of $\omega_\perp$ for measurements made on different days, since small changes in the laser power alter the trap oscillation frequencies. Typically, we obtain a total atom number $N = 2.0(0.2) \times 10^5$ at temperatures $(T/T_F)_{fit} \simeq 0.1$. From the measured trap frequencies, we find (after correction for anharmonicity using Eq. 1 below) $\omega_\perp = \sqrt{\omega_x\omega_y} = 2\pi \times 1596(7)$ Hz, and $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3} = 2\pi \times 560(13)$ Hz. For these parameters, the typical Fermi temperature for a noninteracting gas is $T_F = (3N)^{1/3}\hbar\bar{\omega}/k_B \simeq 2.2\,\mu$K, small compared to the final trap depth of $35\,\mu$K.

Anharmonicity arising from the gaussian trapping potential reduces the measured frequencies ω^{meas} from their zero-energy harmonic oscillator values ω. The transverse oscillation frequency $\omega_\perp$ for the noninteracting gas at 526 G is corrected using the formula [28],

$$\omega_\perp/\omega_\perp^{\mathrm{meas}} = 1 + (6/5)\, M\omega_\perp^2 x_{\mathrm{Brms}}^2/(b_B^2 U), \qquad (1)$$

where U is the trap depth, M is the ^{6}Li mass, and $b_B = \sqrt{1 + (\omega_x t)^2} = 10.3$ is the ballistic expansion factor for the release time $t = 1$ ms. Note that the rms width for the ballistically expanding gas is $x_{\mathrm{Brms}} = \sigma_{TF}^B/\sqrt{8}$,

where σ_{TF}^B is the transverse T-F radius of the ballistically expanded cloud at 526 G.

The measured hydrodynamic frequencies ω_H are corrected for anharmonicity using [28]

$$\omega_H/\omega_H^{\mathrm{meas}} = 1 + (32/25)\sqrt{10/3}\, M\omega_\perp^2 x_{\mathrm{rms}}^2/(b_H^2 U). \quad (2)$$

Here, b_H is the hydrodynamic expansion factor [2, 29], 11.3 after 1 ms, and $x_{\mathrm{rms}} = \sigma_{TF}/\sqrt{8}$ is the rms width for the interacting gas after expansion. Note that b_H rather than b_B is used to estimate the trapped cloud rms radius because we observe anisotropic expansion of the gas [2] which is in good agreement with the predictions of hydrodynamics [2, 29].

The extracted ratios of the corrected frequencies, $\omega_H/\omega_\perp$, and the measured damping ratios $1/(\omega_\perp\tau)$ are given in Table 1 as a function of magnetic field. The frequencies are plotted in Fig. 1 as a function of magnetic field. The lower horizontal scale gives $1/(k_F a)$, where k_F is the Fermi wavevector at the trap center $\hbar k_F = \sqrt{2M\,k_B T_F}$ and $a = a(B)$ is the scattering length. Note that we assume that the Feshbach resonance is located at 834 G, consistent with the best recent estimates [30, 31]. The solid curves in Fig. 1 show the predic-
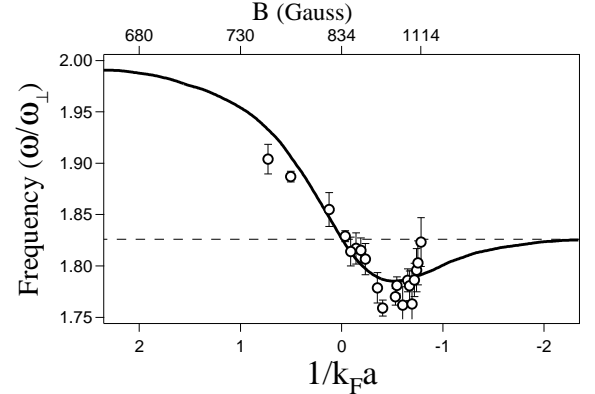

FIG. 1: Magnetic field dependence of the frequency ω of the radial breathing mode. Solid line is the theory based on superfluid hydrodynamics from Hu et al. [17]. Dashed line is the hydrodynamic frequency for a unitarity gas, $\sqrt{10/3}$, as predicted at resonance. Note that the top (magnetic field) axis is not linear.

tions for the frequency by Hu et al. based on assumptions of superfluid hydrodynamics [17, 32].

The data exhibit several interesting features. For fields near resonance, the measured radial breathing mode frequencies are in very good agreement with hydrodynamic theory [15, 16, 17, 18, 19], confirming our previous measurements [13]. Further, the measured breathing mode frequency at 840 G, $\omega_H/\omega_\perp = 1.829(.006)$, is very close to that predicted for a unitary-limited, hydrodynamic Fermi gas, where $\omega_H/\omega_\perp = \sqrt{10/3} = 1.826$. Fig. 2 shows that the damping rates are small near resonance. The maximum damping time of 7 ms, obtained near resonance, corresponds to a minimum damping ratio

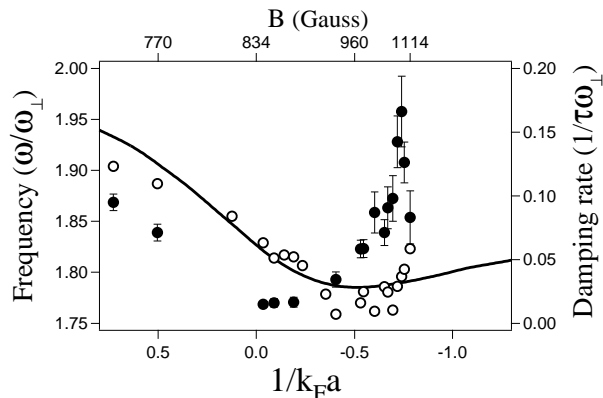$1/(\omega_\perp \tau) = 0.014$, or about 20 periods of hydrodynamic oscillation.

FIG. 2: Magnetic field dependence of the damping rate $1/\tau$ of the radial breathing mode. Damping rate (solid circles, axis labelled on the right); frequency (data–open circles, theory–solid curve, axis labelled on the left). Solid curve (predicted frequency). The maximum damping ratio occurs at 1080G and exceeds the maximum 0.09 permitted for collisional dynamics by a factor of two. Note that the top (magnetic field) axis is not linear.

Away from resonance, the data in Fig. 1 exhibit qualitative agreement with the hydrodynamic theory of Hu et al. [17]. However, the frequencies measured well below resonance, at 750 G and 770 G, are lower than predicted by approximately two standard deviations. Further, above resonance, the measured magnetic field dependence of the frequency is compressed. In addition, there is a region where the data are consistently below the theory and this is followed by a region where the data are consistently above. The frequency reaches its lowest value at 925 G, dropping below the theory by three standard deviations, and then it rises rapidly at 1080 G. At the highest field achievable with our magnets, 1114 G, the frequency is well above the theoretical prediction, but does not reach the asymptote, $2\,\omega_x$, for a noninteracting gas. In this region above resonance, we observe a clear breakdown of hydrodynamics, both in the rapid change in frequency at 1080 G and in the damping rate. A transition is obvious in Fig. 2, which shows the damping rate as a function of B. Near 1080 G, the damping rate increases to $0.17\,\omega_\perp$, well above the maximum $0.09\,\omega_\perp$ attainable by collisional hydrodynamics [21, 22]. The observed behavior is therefore in qualitative agreement with the abrupt frequency change and increased damping observed by the Innsbruck group [14]. However, our results differ from those of Bartenstein et al., in two important respects: i) Near resonance, our observations for the radial breathing mode agree with hydrodynamic theory; ii) the field of 1080 G, at which we observe the complete breakdown of hydrodynamics and rapid damping, is substantially higher than the 910 G value observed by the Innsbruck group [14].

The Innsbruck group has suggested that the break-down may arise when the zero temperature BCS energy gap Δ is smaller than the collective mode energy $\hbar\omega$ [14]. Zubarev [18, 19] has determined the trap averaged value of Δ and shows that if the abrupt transition occurs at 910 G for the Innsbruck group, then for the conditions of our trap, we should observe the transition near 1000 G, assuming 3×10^5 atoms. Using similar estimates, we find that at 910 G, $\Delta \simeq 2\hbar\omega$ for the conditions of the Innsbruck group and $\Delta \simeq \hbar\omega$ for our trap conditions at 1080 G. Since single particle excitations have a minimum energy of 2Δ, perhaps the condition $\Delta = \hbar\omega$ underestimates the required collective mode energy for complete pair breaking. However, it has been noted by Heiselberg [33] that in-gap surface modes have a smaller energy than Δ, but scale similarly, possibly accounting for the observed breakdown at $\Delta > \hbar\omega$.

For the conditions of the Innsbruck group, the breakpoint occurs at the same magnetic field of 910 G at a shallow trap depth and for a trap depth increased by a factor of 9 [14]. This magnetic field independence was first explained by the Innsbruck group and arises as follows. In their case (but not in ours) the axial confinement is provided primarily by the bias field magnets, and the ratio of the Fermi energy to $\hbar\omega_\perp$ and hence to $\hbar\omega$ *decreases* with increasing trap depth. Then, using the most recent values of the scattering length [31], we find the increase in k_F in the exponent of the pairing gap compensates for this reduction at 930 G, consistent with their argument.

We can obtain an alternative estimate of the magnetic field at which an abrupt change occurs using an idea due to Falco and Stoof [34]. They suggest that when the Feshbach ($v = 38$) molecular state in the singlet potential has a higher energy than the two-particle Fermi energy (relative to the zero of the triplet potential), the system becomes BCS-like. In this case, the system may not be interacting strongly enough to remain superfluid at the temperatures we achieve. A simple estimate of the breakdown magnetic field is then obtained from

$$\frac{\hbar^2}{Ma^2} = 2\langle \epsilon_F(\mathbf{x}) \rangle. \qquad (3)$$

We assume that near resonance, interactions make the same contribution to the chemical potential of free triplet atoms and to very weakly bound atoms in large singlet molecules. We also assume the energy of the molecular state is $\epsilon_b = \hbar^2/Ma^2$ relative to the zero of energy in the triplet potential, which is valid in the two-body case near the Feshbach resonance. Using the trap averaged local Fermi energy, we obtain $2\langle \epsilon_F(\mathbf{x}) \rangle = (5/4)\,k_B T_F = (5/8)\,\hbar^2 k_F^2/M$, so that $1/(k_F a) = -\sqrt{5/8} = -0.79$. From Table I, we see that at $B = 1080$ G, $1/(k_F a) = -0.74$, in reasonable agreement. Unfortunately, our simple hypothesis cannot explain the insensitivity of the B field value with respect to trap depth observed by Bartenstein et al.: Scaling our Fermi energy down from $2.2\,\mu$K to $1.2\,\mu$K for the conditions of the Innsbruck group, we predict a transition near $B = 970$ G, higher than observed.

B(G)	$1/(k_F a)$	$(\omega/\omega_\perp)$	$1/(\omega_\perp\tau)$
750	0.728	1.904(.015)	0.095(.009)
770	0.504	1.887(.005)	0.071(.007)
815	0.124	1.855(.017)	*
840	-0.034	1.829(.006)	0.014(.001)
850	-0.090	1.816(.014)	0.016(.005)
860	-0.142	1.817(.015)	*
870	-0.190	1.815(.012)	0.017(.004)
880	-0.235	1.807(.015)	*
910	-0.354	1.779(.015)	*
925	-0.405	1.759(.008)	0.034(.006)
969	-0.531	1.766(.007)	0.058(.007)
975	-0.546	1.778(.007)	0.059(.007)
1000	-0.603	1.763(.018)	0.087(.016)
1025	-0.652	1.786(.011)	0.071(.010)
1035	-0.670	1.781(.015)	0.091(.016)
1050	-0.695	1.764(.033)	0.098(.020)
1065	-0.719	1.786(.016)	0.142(.020)
1080	-0.740	1.796(.021)	0.166(.028)
1090	-0.754	1.803(.021)	0.126(.016)
1114	-0.783	1.823(.024)	0.083(.021)

TABLE I: Breathing mode frequencies ω and damping rates $1/\tau$ as a function of applied magnetic field B. Error estimates include the statistical error from the fit only. *Denotes data taken with a 50 μs excitation time [13] for which we omit the damping rate. $a(B)$ is determined from Ref. [31]

In summary, we have measured the magnetic field dependence of the frequencies and damping rates of the radial breathing mode for a strongly interacting Fermi gas of ^{6}Li. The measurements are in very good agreement with the predictions of hydrodynamics near the Feshbach resonance at 834 G, but we observe a breakdown of hydrodynamics away from resonance. Beginning at higher fields near 925 G we observe first a frequency decrease, well below predictions, and then an abrupt frequency increase at a field of 1080 G, accompanied by rapid damping, which exceeds the maximum damping for a collisional normal fluid.

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